I'm Thinking differently about...

- Helping students read math problems
- The way I teach my students to diagram
- Scaffolding
- Quantities and relationships, i.e. how to be more precise about them
- How to introduce problem solving at the beginning of the year, i.e. through routines
- PD for my district. It should be focused on instructional routines vs. products/curriculum

I now understand...

- A quantity is anything in a problem you can count or measure, and does not need to be associated with a specific value
- Quantities and relationships are very different
- Not all of the MPs hold the same weight, some support others
- Using sentence starters are a great way to reinforce vocab for ELLs and SWLDs

A Question I have...

- How long does a routine take? Does an intro lesson differ from a 5th or 6th in terms of length?
- Do you have any templates/student materials?
- Where to find/ how to select tasks for routines?
- How common is it for teachers to use these routines?
- Do you change the routine based on grade level?
- How to differentiate for students who are able to complete these tasks quickly and adeptly?

I want you to know...

- I love the "topic sentence" and "details" visualsvery helpful.
- I'm still wondering how ELL's experience math practices
- Great pacing today

Fostering the Mathematical Practices in English Learners and Students with Learning Disabilities

Grace Kelemanik Amy Lucenta



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Share...discuss...reflect..

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Goal # 1

Understand how developing mathematical practices supports special populations. You will know your learning is on track if you can:

• Describe how the math practices align with best practices for supporting students with learning disabilities and English learners.

Goal # 2

Learn the Instructional Routines, Connecting Representations and Recognizing Repetition. You will know your learning is on track if you can:

- Articulate the flow of the Instructional Routines
- Describe how each routine provides on-ramps and continued support for all learners to 'think like mathematicians'

Agenda

- Opening Goals and Agenda
- MPs and SpPops: A Symbiotic Relationship
- Connecting Representations Instructional Routine Deep Dive
- Lunch
- Recognizing Repetition Instructional Routine Deep Dive
- Select a 'Focus' Routine
- Wrap Up and Adjourn



MATH PRACTICES

a. An essential goal for all?

b. A critical support for special populations?.

















COMMUNICATE IDEAS MP1 MP3 MP6





CONNECTIDEAS & REPRESENTATIONS MP2 MP4 MP5 MP7











A SYMBIOTIC RELATIONSHIP

SUPPORT SPECIAL POPULATIONS

TEACH MATH PRACTICES AUTHENTICALLY SWLD LEI Work within contexts pportunities to communicate ideas **Multisensory learning experiences Connect ideas & representations Opportunities for multiple strategies** Multiple practice opportunities for students to develop mathematical understanding Carefully planned range & sequence of examples



SWLD LEI Work within contexts pportunities to communicate ideas **Multisensory learning experiences Connect ideas & representations Opportunities for multiple strategies** Multiple practice opportunities for students to develop mathematical understanding Carefully planned range & sequence of examples

HOW ARE INSTRUCTIONAL ROUTINES SUPPORTS for SPECIAL POPULATIONS?



So...an instructional routine that focuses on one or more avenues of thinking, and has 'baked-in', research-based supports for special populations...



Quantitative Reasoning supports ALL students....especially

- Students who don't know where to begin to solve a word problem
- Students who struggle with multi-step problems
- Students who benefit from working within contexts
- Students who benefit from drawing/using visual representations
Structural Thinking supports ALL students....especially

- Get lost in details or tedious calculations
- Benefit from visual representations
- Benefit from connections between and among math ideas & representations
- Interpret the 'big picture' or shift perspective

Repeated Reasoning supports ALL students....especially

- Students who benefit from multiple modalities
- Students who struggle to abstract and generalize
- Students who work in organized and/or systematic ways
- Students who benefit from seeing how rules are developed

Developing mathematical practices requires high leverage teaching practices

> Instructional Routines embody NCTM effective teaching practices

and equitable practices for students, teachers, and districts.



Figure 1.1 Diagram that shows the relationship of the practices to each other

Connecting Representations

An Instructional Routine to Develop ALL Students' Structural Thinking

Routines for Reasoning Kelemanik, Lucenta, & Creighton





Connecting Representations

WHAT: Match visuals to expressions by chunking, changing the form, and connecting to math you know

WHY: To "think like mathematicians", to use mathematical *structure* to match two different representations.





Ask yourself...

What part of the visual will help me connect to a chunk of the expression?

What about the expression will help me connect to the visual?





Create a Representation

Ask yourself...

"What do you notice about this expression?"

THINK

"How can you chunk this expression into pieces you can describe?"

Create a Representation



- Share your interpretations of the expression.
- Together create a matching visual representation.



Share

They noticed... so they...

When they saw...it made them think of... so they...



Meta-Reflection

000

A. When interpreting an *expression / visual*, I learned to pay attention to...

B. When connecting representations, I learned to ask myself...

C. A new mathematical connection I made is...

Reflect on CR Instructional Routine



What stands out to you about the Connecting Representations instructional routine?

What questions do you have?

Connecting Representations Container



Kelemanik, Lucenta, & Creighton

MP7 in Connecting Representations

MP7 Goal

- To learn to "think like a mathematician" by connecting two things that look nothing alike at all, but represent the same underlying structure.
- Task selection that invites structural thinking
- Questions that orient attention to structure
 - What are the pieces of the visual and how do they connect to the rule?
 - How can you make sense of those pieces?
 - What do those operations imply?
- Meta-cognitive reflection focused on structure
 - I noticed.....so, I looked for....
 - reminded me of _____

Connecting Representations

WHAT: Match words (verbal descriptions) to expressions by chunking, changing the form, and connecting to math you know

WHY: To "think like mathematicians", to use mathematical *structure* to match two different representations.





Ask yourself...

What chunk of the verbal description will help me connect to a part of the expression?

What about the expression will help me connect to the verbal description?





Create a Representation

Ask yourself...

"What do you notice about this expression?"

THINK

"How can you chunk this expression into pieces you can describe?"

Create a Representation



- Share your interpretations of the expression.
- Together write a matching verbal description.



Share

They noticed... so they...

When they saw...it made them think of... so they...



Meta-Reflection

000

A. When interpreting an expression, I learned to pay attention to...

B. When connecting representations, I learned to ask myself...

C. A new mathematical connection I made is...



Routines for Reasoning Kelemanik, Lucenta, & Creighton

Connecting Representations Container



Routines for Reasoning Kelemanik, Lucenta, & Creighton



Reflect on CR Instructional Routine



How does/can the Connecting Representations instructional routine provide access and support for SWLD to develop the math practices?

Essential Strategies

Next time I will.... before I

calculate because

Annotation



Four Rs: Repeat, **Q** Rephrase, Reword, Rec

Ask Yourself Questions

Baked-In Supports for Students with Learning Disabilities within Connecting Representations

• Provide **multiple passes** at articulating the underlying structure between representations using the **Four Rs**.

 Reference sentence frames and starters to prompt students to articulate observations that sparked the structural thinking and that can be applied again in other math problems.

• Use **annotation** to make structural connections explicit for students and to provide visual residue of the mathematical discussions.

- Choose representations that support students' learning strengths.
- Support students' participation in full-group discussions by defining partner roles.

Routines for Reasoning Kelemanik, Lucenta, & Creighton, p. 86-87

Baked-In Supports for ELLs within Connecting Representations

 Use sentence starters and sentence frames to develop structural language (e.g chunk, change, connect) and help students communicate structural elements of representations.

 Make use of the Four Rs to provide multiple opportunities to develop and refine academic language crucial for describing structural connections among representations.

 Ensure that gestures and annotation support the structural ideas and language and thinking that is being communicated verbally.

Routines for Reasoning Kelemanik, Lucenta, & Creighton, p. 86-87

Consider Sophia: How could the routine and MP7 support her?

Sophia has difficulty persevering in a problem situation as she often perseverates on her initial idea and becomes frustrated. She benefits from visual representations because they help her remember concepts and experiences. When provided multiple examples over time, Sophia is able to execute algorithms with accuracy and efficiency.

The Power of Routines

Instructional routines...

- support students.
- are collaborative.
- free up brain space for the hard work.
- serve as vehicles for the Five Practices.
- save time.
- develop math practices over time.
- Wash, rinse, repeat.....AND....
- Develop equitable practice in a classroom, school, district.




Figure 1.1 Diagram that shows the relationship of the practices to each other

An Instructional Routine to Develop Repeated Reasoning



Repeated Reasoning Deep Dive Goal

Learn how to teach students to reason through repetition

- Understand how repetition in *process* is different from number patterns
- Know the different types of mathematical "processes" to mine for repetition
- Understand the flow and purpose of the *Repeated Reasoning* Instructional Routine

Recognizing Repetition Deep Dive

- 1. A few words about repeated reasoning
- 2. Experience the *Recognizing Repetition* Instructional Routine x2
- 3. Unpack the *Recognizing Repetition* Instructional Routine
- 4. Consider who this routine supports and how it supports them

Repeated Reasoning (MP8)

Attend to...

Repetition in **Processes**

Counting Calculating Constructing

Ask yourself...

- Do I keep doing the same thing over and over again?
- What about the process is repeating?
- How can I generalize the repetition?
- Have I included every step?

Repeated Reasoning (MP8)

Actions You Take...

- Count in an organized way
- Draw or build several figures
- Try several numbers and observe the process
- Record and track calculations
- Generalize the repetition
- Simultaneously maintain oversight of the process while attending to details
- Monitor and evaluate reasonableness of intermediate results

Decompressing Repeated Reasoning

- Pay attention to the process
- Sense the regularity
- "Shortcut" the process
- Connect the process to an "input" value
- Generalize the process to a rule



An Instructional Routine to develop the repeated reasoning avenue of thinking



Routines for Reasoning

Kelemanik, Lucenta & Creighton

Container





WHAT: Sense the repetition in the way you draw/build and generalize the repetition.

WHY: To "think like mathematicians", look for regularity in your counting, calculating, and *constructing* processes.







Notice Repetition Draw or Build

Ask yourself...

Am I drawing/building the same way each time?





Ask yourself...

Are they drawing/building the same way each time?









Ask yourself...

How can I use the repetition in my drawing/ building process to find the number of circles in figure 10?





number of circles in figure N.





Ask yourself...

What is the repetition they are generalizing?

The repetition they generalized is...





Meta-Reflection

000

A. When looking for repetition, I learned to pay attention to...

B. When generalizing repetition, I learned to...





WHAT: Look for regularity in your *calculating* process and generalize that repetition.

WHY: To "think like mathematicians", look for regularity in your counting, *calculating*, and constructing processes.











Notice Repetition In Calculations

Ask yourself...

Am I calculating the same way each time?

Mr. Munroe was buying furniture for the McKay School. He spent \$7,600 on chairs and also bought some tables for \$50 each. Find out the total amount he would spend on furniture if he bought 1, 2, 3, and 10 tables.

Notice Repetition Show Calculation Process

Ask yourself...

Are they calculating the same way each time?









Ask yourself...

How can I use the regularity in my calculations to make a rule for any number of tables?

Mr. Munroe was buying furniture for the McKay School. He spent \$7,600 on chairs and also bought some tables for \$50 each. Find a rule for determining the total amount Mr. Munroe would spend given any number of tables.







Ask yourself...

What is the repetition they are generalizing?

The repetition they generalized was...




Meta-Reflection

000

A. When looking for repetition, I learned to pay attention to...

B. When generalizing repetition, I learned to...

What is it about the *Recognizing Repetition* Instructional Routine that...

- Provides access and support to English learners?
- Provides access and support to students with learning disabilities?



How does Recognizing Repetition Support Special Populations?

- It's predictable!
- Combats learned helplessness because it invites multiple sensing modalities and habitualizes "ask yourself" questions
- Promotes a view of generalizing that is sensible, not magic
- Places a premium on processing time and modalities
- Provides opportunities to develop and practice language

4 Essential Instructional Strategies

Keeping the focus on the mathematical thinking while providing access for a wide range of learners

- Ask-yourself questions
- Annotation
- Sentence frames and starters
- The Four Rs repeat, rephrase, reword, record

Repeated Reasoning "Look Fors"

Your students know that:

- Repetition in their counting, calculating, or building can be seen, heard, and felt.
- They can use regularities in their process to identify a generalized statement about the mathematics.
- A key to generalizing repetition is to connect an aspect of the repetition in their process to a quantity in the problem (such as the term of the series, the tower number, or one of the variables in the problem).
- If there is no repetition in a problem situation, they can sometimes create repetition by trying numbers and tracking their calculation process.

Your students *regularly* take the following actions:

- Pay attention to how they count, calculate, and construct.
- Record their counting, calculating, and constructing processes and look for repetition in them.
- Show examples of using their senses to highlight repeated reasoning; for example, they might recite repetitive steps they're taking, move manipulatives in a rhythmic way through a process, draw a diagram that shows some repeating process, physically act out a process to notice what repeats, etc.



Consider Chantal: How could the routine and MP8 support her?

Chantal is able to discern key concepts and ideas when provided with time to process the ideas. However, she loses track of details in reading as well as mathematics. Chantal benefits from mnemonic devices and graph paper in order to implement algorithms successfully. Chantal has difficulty organizing her materials, and has found success when her notebooks are color-coded, and she is provided with systems to keep track of her belongings.



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Next Steps:

Select a routine to plan for during tomorrow's session

- Three Reads
- Capturing Quantities
- Connecting Representations
- Recognizing Repetition

Complete Google Form via website link (Related Resources Tab - Presentations)

Read the corresponding chapter in the book

Feedback



3 Things I learned

2 Questions I have

1 Thing I want you to know

Homework



- Read the chapter in Routines for Reasoning on the routine you selected to try
 - Chapter 3 Capturing Quantities (MP2)
 - Chapter 4 Connecting Representations (MP7)
 - Chapter 5 Recognizing Repetition (MP8)
 - Chapter 6 Three Reads (MP1)