Teaching
for Thinking

Fostering Mathematical
Teaching Practices
Through Reasoning
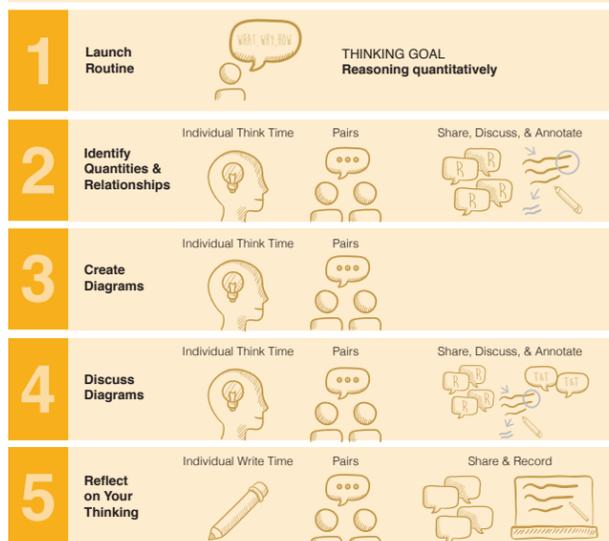Routines

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Amy Lucenta

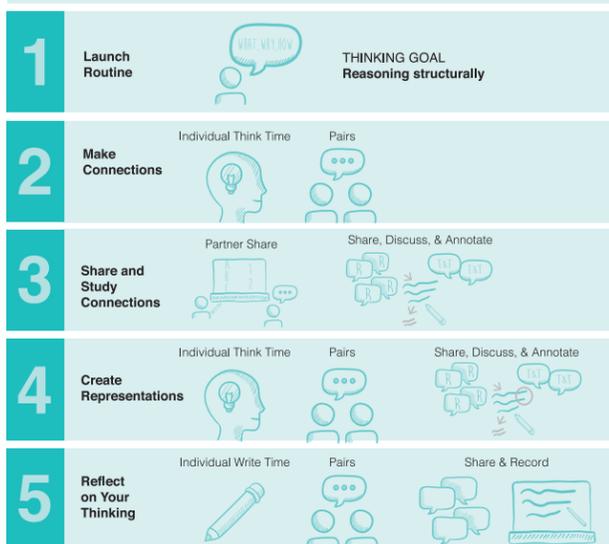## Four Reasoning Routines from *Routines for Reasoning*

Each routine targets a specific standard for mathematical practice, but they all share common design features that keep a laser focus on math thinking and provide access to a wide range of learners. Each reasoning routine includes a brief description and visual that shows the repeatable flow. Icons within each visual highlight the designs for interaction—how students engage with each other, the content, and the teacher—that remain the same within each routine.

Capturing Quantities

1 Launch Routine — THINKING GOAL: Reasoning quantitatively
2 Identify Quantities & Relationships — Individual Think Time / Pairs / Share, Discuss, & Annotate
3 Create Diagrams — Individual Think Time / Pairs
4 Discuss Diagrams — Individual Think Time / Pairs / Share, Discuss, & Annotate
5 Reflect on Your Thinking — Individual Write Time / Pairs / Share & Record

The goal of the Capturing Quantities reasoning routine is to develop quantitative reasoning (Common Core State Standards [CCSS] Standards for Mathematical Practice [SMP] 2, Reason abstractly and quantitively [National Governors Association Center for Best Practices, Council of Chief State School Officers 2010]). In the routine, students identify quantities and relationships in a problem situation, work with a partner to create a diagram that shows all the quantities and the relationships among them, then share diagrams in the full group and discuss where/how they see various quantities and relationships in the diagrams. Finally, students reflect on what they have learned about reasoning quantitatively.

Connecting Representations

1 Launch Routine — THINKING GOAL: Reasoning structurally
2 Make Connections — Individual Think Time / Pairs
3 Share and Study Connections — Partner Share / Share, Discuss, & Annotate
4 Create Representations — Individual Think Time / Pairs / Share, Discuss, & Annotate
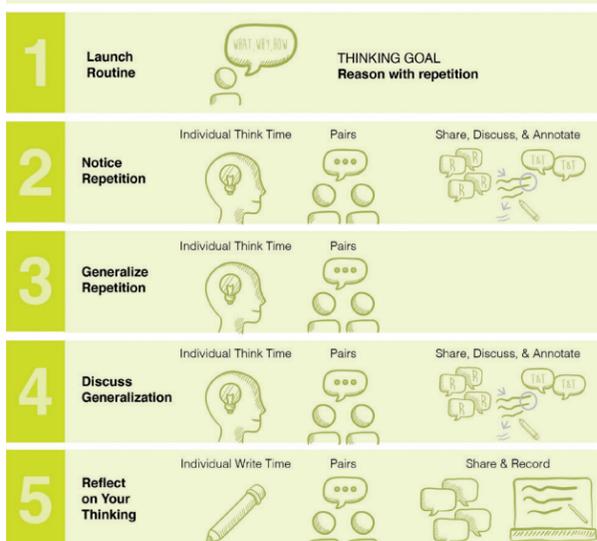5 Reflect on Your Thinking — Individual Write Time / Pairs / Share & Record

The goal of the Connecting Representations reasoning routine is to develop structural thinking (CCSS SMP 7, Look for and make use of structure). In the routine, students analyze and then work with a partner to connect two different types of representations, share and study connections in the full group, then create, share, and discuss a missing representation. Finally, students reflect on what they have learned about thinking structurally. You saw a glimpse of this routine in Mr. Ryan's class in Chapter 1.
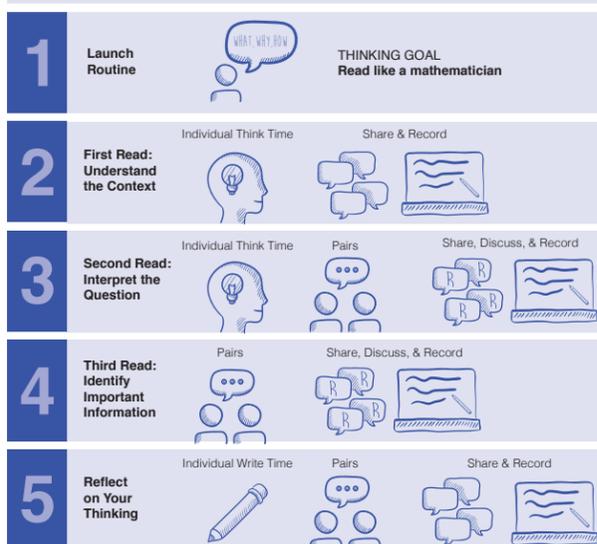
Figure 2–1

Recognizing Repetition

1 Launch Routine — THINKING GOAL: Reason with repetition
2 Notice Repetition — Individual Think Time / Pairs / Share, Discuss, & Annotate
3 Generalize Repetition — Individual Think Time / Pairs
4 Discuss Generalization — Individual Think Time / Pairs / Share, Discuss, & Annotate
5 Reflect on Your Thinking — Individual Write Time / Pairs / Share & Record

The goal of the Recognizing Repetition reasoning routine is to develop repeated reasoning (CCSS SMP 8, Look for and express regularity in repeated reasoning). In the routine, students engage in a counting, constructing, or calculating process and sense the regularity, share repetitions they are noticing, and then work with a partner to generalize the repetition. They then share and discuss generalizations in the full group, and finally, they reflect on what they have learned about reasoning through repetition.

3 Reads

1 Launch Routine — THINKING GOAL: Read like a mathematician
2 First Read: Understand the Context — Individual Think Time / Share & Record
3 Second Read: Interpret the Question — Individual Think Time / Pairs / Share, Discuss, & Record
4 Third Read: Identify Important Information — Pairs / Share, Discuss, & Record
5 Reflect on Your Thinking — Individual Write Time / Pairs / Share & Record

The goal of the Three Reads reasoning routine is to make sense of math problems (CCSS SMP 1, Make sense of problems and persevere in solving them). In the routine, students read a word problem three times, each time for a different purpose. After the first read, students make sense of the context, answering, "What's the problem about?" After the second read, pairs restate the question in their own words, and share and discuss their rephrased questions. After the third read, students identify important information and as a group share and record information. Finally, students reflect on what they've learned about reading and interpreting a math problem.
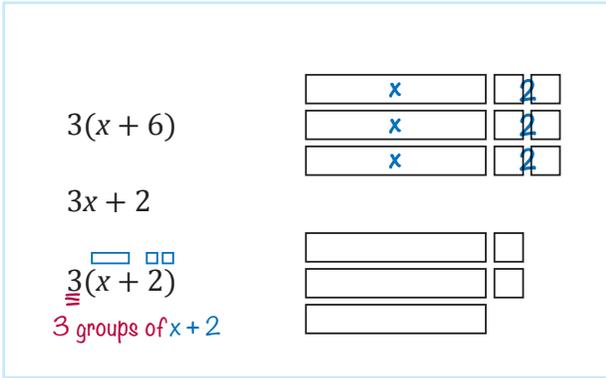
$3(x + 6)$
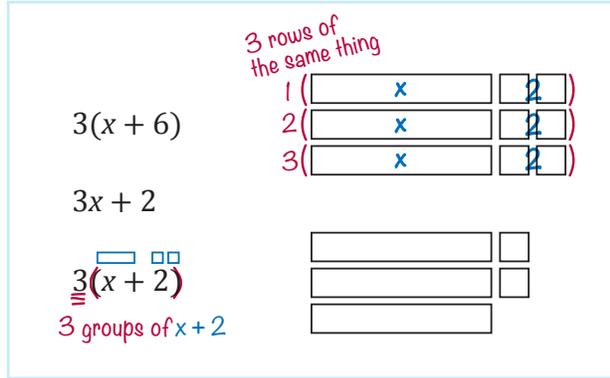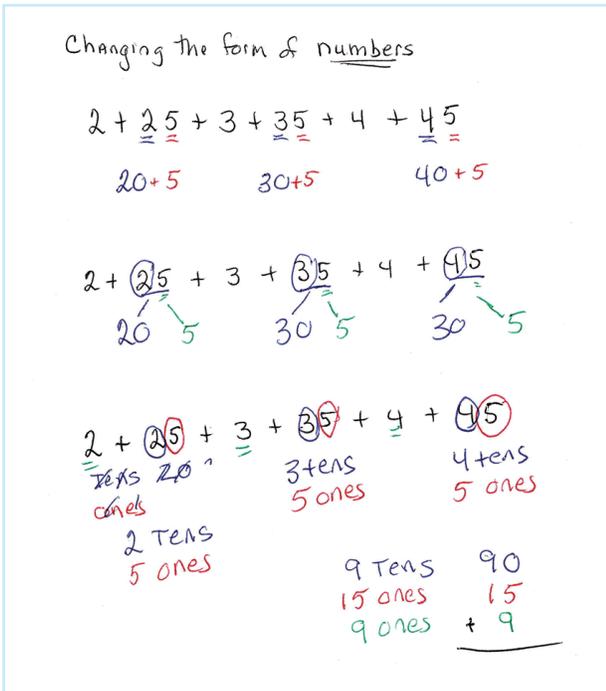
$3x + 2$

$\underline{3(x + 2)}$

3 groups of $x + 2$

Figure 2–3

3 rows of the same thing

$3(x + 6)$

$3x + 2$

$\underline{3(x + 2)}$

3 groups of $x + 2$

Figure 2–4

Changing the form of numbers

$2 + \underline{25} + 3 + \underline{35} + 4 + \underline{45} =$

$20 + 5 \qquad 30 + 5 \qquad 40 + 5$

$2 + 25 + 3 + 35 + 4 + 45$

$20 \quad 5 \qquad 30 \quad 5 \qquad 30 \quad 5$

$2 + 25 + 3 + 35 + 4 + 45$

tens 20

ones

2 Tens
5 ones

3 tens
5 ones

4 tens
5 ones

9 Tens   90
15 ones   15
9 ones   + 9

Changing the form of the expression

$2 + 25 + 3 + 35 + 4 + 45$

$25 + 35 + 2 + 3 + 45 + 4$

$60 \quad + \quad 50 \quad + 4$

$2 + 25 + 3 + 35 + 4 + 45$

25+5

$30 \qquad\qquad 80 \qquad 4$

Changing the form of both #'s And Expressions

$2 + 25 + 3 + 35 + 4 + 45$

$20 + 30 + 40 + 5 + 5 + 5 + 2 + 3 + 4$

$90 \qquad\qquad 20 \qquad 4$
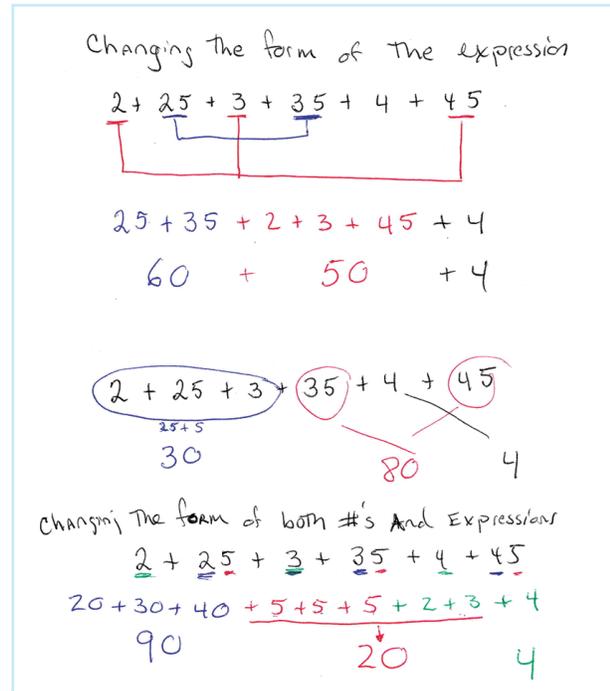
Figure 2–5

Examples of annotations. Consider use of color, words, symbols, visuals, and so on.

Decide and Defend

1	**Launch Routine**	WHAT, WHY, HOW	THINKING GOAL **Critique and construct arguments**	

		Individual Think Time	Pairs	Share, Discuss, & Record
2	**Interpret the Work**			

		Individual Think Time	Pairs	Annotate
3	**Decide if it is Correct**			

		Pairs	Annotate
4	**Draft Your Defense**		

		Partner Share	Discuss & Annotate
5	**Defend Your Decision**		

		Individual Write Time	Pairs	Share & Record
6	**Reflect on Your Thinking**			

Figure 3–5

Figure 3–6

Figure 3–7   Hugo and Daniella

Surface area
example from
Mr. Driscoll's class
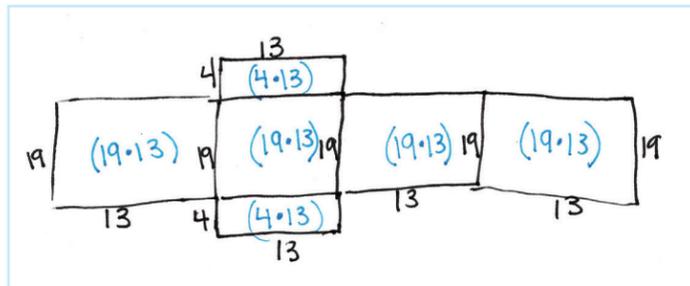
Figure 3–8   May and Michael
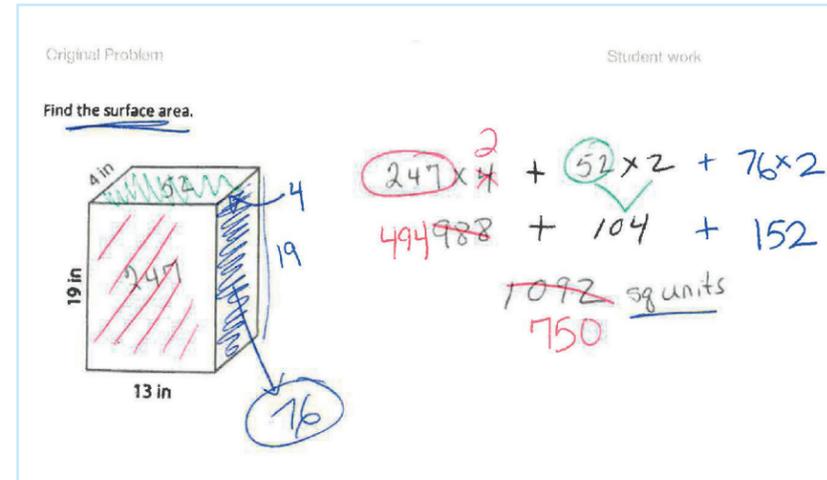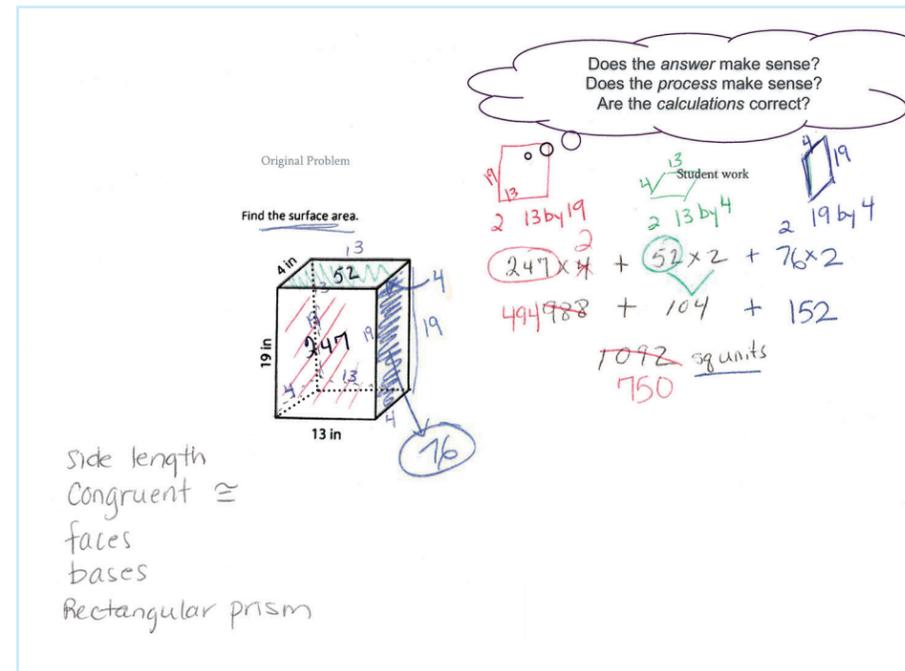

Figure 3–9   Marco and Vanessa

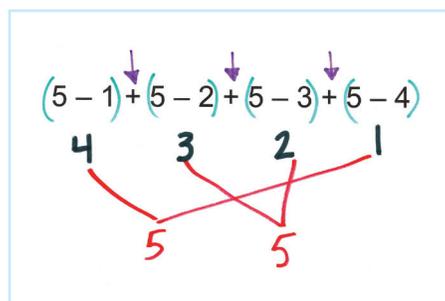Figure 3–10   Mr. Driscoll's annotated version of Marco and Vanessa's work
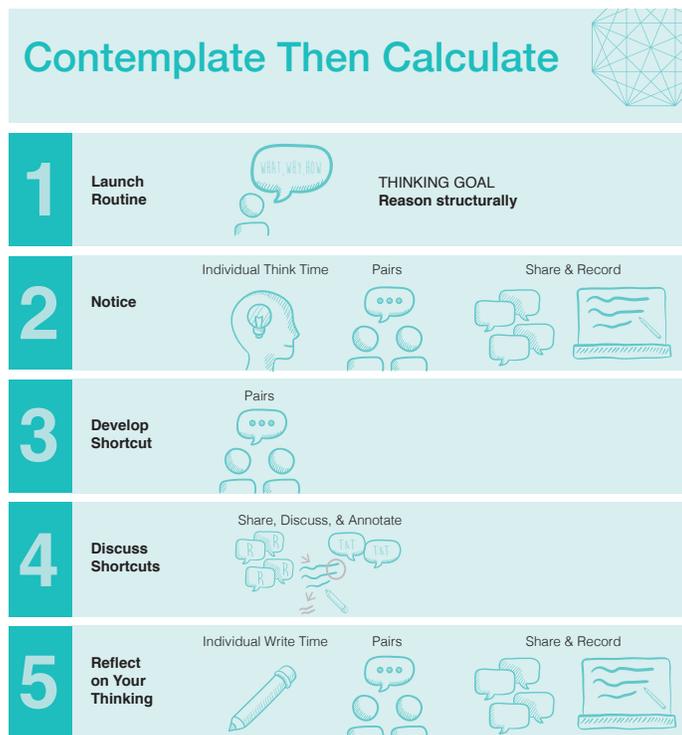
Figure 4–9

Figure 4–10   **Example of an annotation**
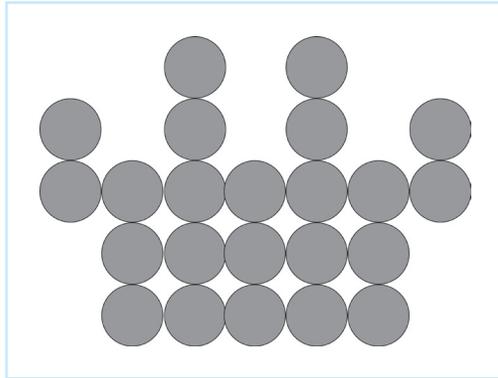
Figure 4–11

Figure 4–12

The dot exercise

Figure 4–13

Analyzing Contexts & Models

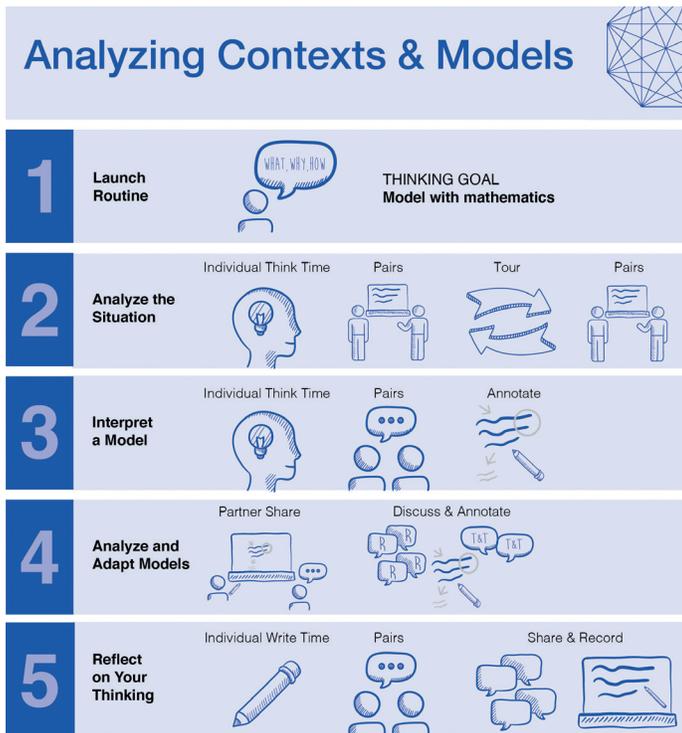|					
1	**Launch Routine**	WHAT, WHY, HOW	THINKING GOAL **Model with mathematics**		
2	**Analyze the Situation**	Individual Think Time	Pairs	Tour	Pairs
3	**Interpret a Model**	Individual Think Time	Pairs	Annotate	
4	**Analyze and Adapt Models**	Partner Share	Discuss & Annotate		
5	**Reflect on Your Thinking**	Individual Write Time	Pairs	Share & Record	
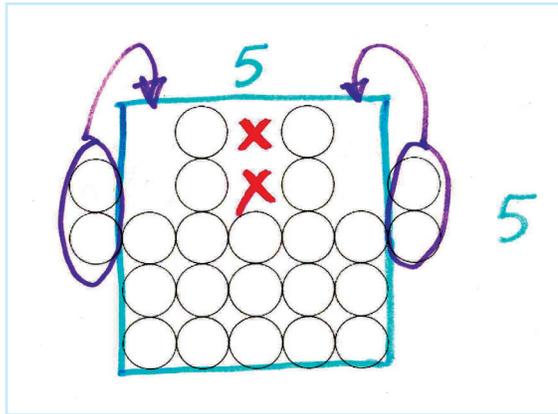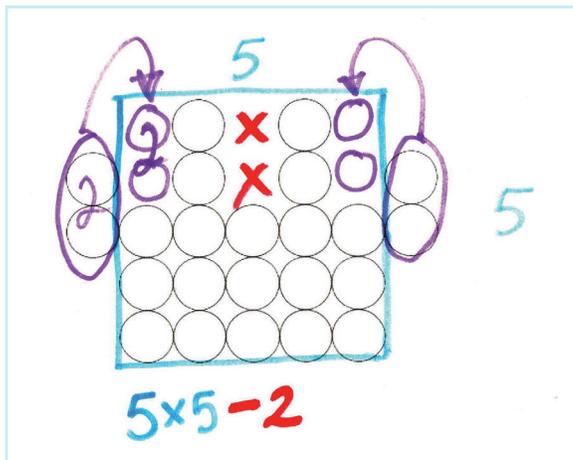
Figure 5–4

Step 1: Launch the routine.

Analyzing Contexts and Models

WHAT: Consider the mathematics of a real world situation, and analyze a model that represents the situation

WHY: To interpret and engage in the real world with a mathematician's eye. To understand how quantities and relationships from a context are represented in a math model.

[A]

Analyzing Contexts and Models

Make Sense | Analyze the Situation | Interpret the Model | Analyze and Adapt Models | Reflect on Learning

[B]

Step 2: Analyze the situation.

Make Sense of the Situation

Ask Yourself:

- What's the question I'm exploring?
- What about the context do I need to consider?

Insert Situation

[C]

Analyze the Situation

Pair | Tour | Pair

[D]

Share Interpretations & Analyze the Situation

Standing with your partner, share your interpretations of the context and together create a list:

- Important quantities are…
- It will be helpful to know…

Insert Situation

[E]

Consider Classmates' Interpretations

Tour the room and read others' lists. Consider what you'd add to your own.

Ask yourself:
- Have they considered something we should also consider?
- Have they (or we) made assumptions?

Insert Situation

[F]

Consider Classmates' Interpretations

Return to your chart, reflect and refine

- Place a + next to key ideas
- Place a - next to ideas less relevant
- Describe quantities as 'The number/amount of…'
- Articulate questions as quantities 'How much/many…?'

Insert Situation

[G]

Step 3: Interpret the model.

Interpret a Model

[H]

Interpret a Model

Ask yourself:
- Where/how does the model represent quantities?

Insert Model

[I]

Interpret a Model

Share the quantities with your partner, together identify questions you have and assumptions the model makes.

They considered the number/amount of…

They found the number/amount of… by…

A question I have about the model is…

An assumption the model makes is….

Insert Model

[J]

Step 4: Analyze and adapt the model.

Analyze and Adapt the Model

- We think the model predicts … because…
- We aren't sure the model is precise because…
- The estimation impacts the outcome because…

Insert Annotated Partners' Interpretation of the Model

[K]

Step 5: Reflect on your thinking.

Reflect on learning

- Next time I consider a situation and try to mathematize it I will ask myself….
- When thinking about constraints, it's important to….
- When analyzing models, I learned to pay attention to….
- A critical feature of modeling is….

[L]

Figure 5–6 **Sample slides**

Place an *x* along the range that best captures your experience with each essential strategy.
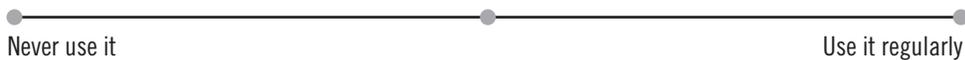
1. Annotation

Never use it   Use it regularly

2. Ask-yourself questions

Never use it   Use it regularly

3. Four Rs: repeat, rephrase, reword, record

Never use it   Use it regularly

4. Sentence frames and starters

Never use it   Use it regularly

5. Turn-and-talks

Never use it   Use it regularly
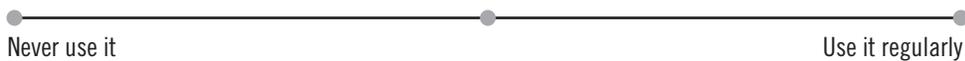
Figure 6–1   **Survey**

Practicing Annotation Offline

Sample tasks accompanied by student shortcuts that might occur in Contemplate Then Calculate. Think about the aspect of structural thinking you would like to highlight and write that down in the space provided for the structural thinking goal. Practice annotating the task multiple times. Reflect on the ways in which your annotation helps students see the structural thinking and how closely the annotation hews to the student thinking.

1. Structural Thinking Goal: _____

   Student thinking: "We noticed the symmetry and the columns of 2, so we got 6 and 1 more for the left side, doubled that, and then added the 2 in the middle."
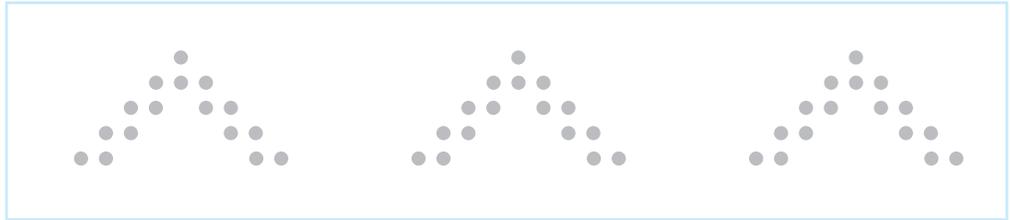

Figure 6–5

2. Structural Thinking Goal: _____

   Student thinking: "We knew that 40% is $\frac{2}{5}$, and there are five 7s in 35, so 2 of them would be 14."

   40% of 35   40% of 35   40% of 35

Figure 6–6

3. Structural Thinking Goal: _____

   Student thinking: "We noticed the pattern 2, 3, 4 and the 20, 30, 40 and that made us think of place value, so we added the 10s and got 90 and we added the 1s and got 5, 10, 15 plus 9 is 24, and just put the 24 and the 90 together, and got 90, 100, 110, 114."
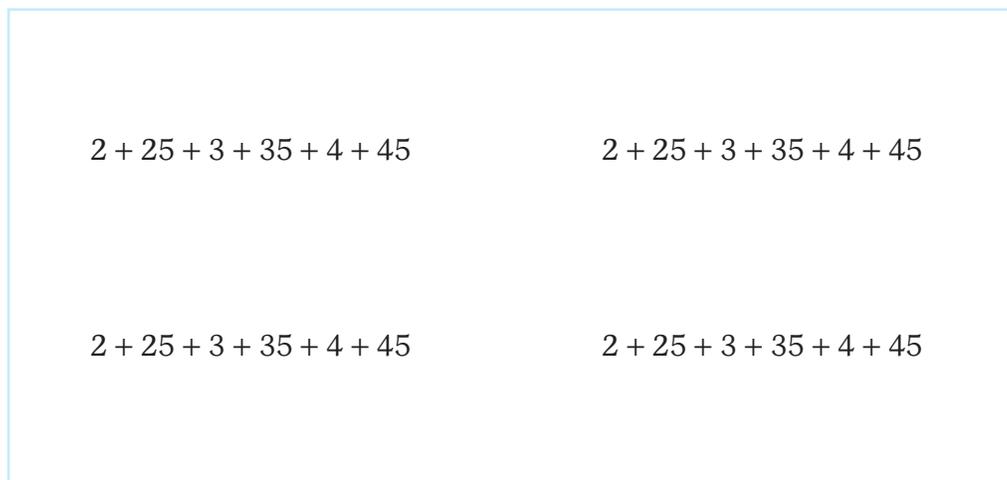
   2 + 25 + 3 + 35 + 4 + 45   2 + 25 + 3 + 35 + 4 + 45

   2 + 25 + 3 + 35 + 4 + 45   2 + 25 + 3 + 35 + 4 + 45

Figure 6–7